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INDIRECT DETECTION OF DARK MATTER WITH ANTIMATTER: DEMYSTIFYING THE CLUMPINESS BOOST FACTORS

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Abstract

The hierarchical scenario of structure formation, in the frame of the Λ-CDM cosmology, predicts the existence of dark matter (DM) sub-halos down to very small scales, of which the minimal size depends on the microscopic properties of the DM. In the context of annihilating DM, such substructures are expected to enhance the primary cosmic ray (CR) fluxes originating from DM annihilation in the Galaxy. This enhancement has long been invoked to allow predictions of imprints of DM annihilation on the antimatter CR spectra. Taking advantage of the method developed by [2], we [1] accurately compute the boost factors for positrons and anti-protons, as well as the associated theoretical and statistical errors. To this aim, we use a compilation of the latest results of cosmological N-body simulations and the theoretical insights found in the literature. We find that sub-halos are not likely to significantly boost the exotic production of antimatter CRs.

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Introduction: Like other topics in fundamental physics, annihilating dark matter is rather well motivated by some *coincidence* arguments, because microscopic new physics is expected to surge at the electroweak energy scale: This energy scale gives also the right interaction amplitudes to naturally get the correct abundance of WIMPs today to account for the cosmological DM energy density, provided that they are thermally produced in the early Universe, without matter-antimatter asymmetry (for a recent review on dark matter candidates, see the nice lecture by [3]). The dark matter annihilation properties are intimately connected to its possible direct interaction with normal matter, and this sketches two complementary approaches in order to hunt for dark matter signatures, beside its production in particle colliders. Direct detection experiments focus on energy deposits on very sensitive and deep underground detectors, while indirect detection aims at observing annihilation traces in the form of neutral or charged cosmic rays (for a review on indirect detection, see e.g. [4]). Primary antimatter CRs originating from DM annihilation could be hunted e.g. in the positron, anti-proton and anti-deuteron spectra at the Earth [5]. In particular, the positron spectrum has some features around 10 GeV that are still hardly understandable from a secondary origin [6]. Some authors tried to explain this with DM annihilation induced positrons, but most of the predictions rely on a rescaling of the primary fluxes by invoking some constant *boost factors* due to DM clumpiness (see e.g. [8, 9]). Indeed, many DM substructures are predicted in CDM-type cosmologies, and as the annihilation rate is proportional to the squared DM density, it naturally increases in an inhomogeneous medium, for a given average density. Though the idea of enhancement due to sub-halos is far from new [7], it has never been fully addressed in the context of charged cosmic rays, contrary to gamma-rays, due to the additional difficulty of dealing with CR propagation. Furthermore, because dark matter clumps might be numerous and countable (at least theoretically or in simulations), they are better described by some phase space distributions. The full treatment of this problem has therefore to make use of statistics, and should also provide either predictions and the associated statistical uncertainties (beside the theoretical ones). As a matter of fact, from an observational point of view, any signature can be said unambiguous if, at least, the theoretical predictions do not suffer from large uncertainties, unless the expected signature is very specific and clear. [2] developed a detailed method in order to translate the clumpiness phase space in terms of probability for the primary signal/boost. These authors focused on positrons, and took a very simple model in which all sub-halos had the same properties and were spatially distributed according to the host DM halo profile. They showed that the boost factor and the statistical uncertainties actually non trivially depends on the detected CR energy. We have used their method, which is quite general and applies to any CR species, but with more sophisticated and more precise inputs for the dark

matter distribution. This proceeding summarises the work presented in [1], to which we refer the reader for a more exhaustive bibliography, and in which we accurately compute the boost factors resulting from dark matter clumpiness for positrons ($e^{+\prime}$s) and anti-protons ($\overline{p}'$s).

Dark Matter distribution in the Galaxy: Any prediction for indirect detection of DM strongly depends on the DM distribution that is used, for the smooth halo as well as for sub-halos (we refer to the *smooth* dark matter component as the host DM halo of the Milky Way — MW). The density profile of dark objects is the main quantity to anticipate how the DM annihilation rate may spatially evolve in the Galaxy. Though axysymmetry characterises most of the results of cosmological N-body simulations, we adopt here a spherical symmetry to describe the Galactic host halo and the sub-halos. A structure of any scale may thus have its density profile $\rho(r)$ depicted by the generic formula proposed by [10]: $\rho(r) = \rho_s / \left[(r/r_s)^\gamma \left[1 + (r/r_s)^\alpha\right]^{(\beta-\gamma)/\alpha}\right]$, where ρ_s is twice the density at the scale radius r_s, at which the logarithmic slope changes from γ to $(\beta-\gamma)/\alpha$. The well-known NFW [11] and Moore [12] profiles are recovered with $\{\alpha, \beta, \gamma\} = \{1, 3, 1\}$ and $\{1.5, 3, 1.5\}$ respectively. An NFW behaves like r^{-1} while a Moore scales like $r^{-1.5}$ in the central region of the structure. The scale radius is connected to the *concentration* parameter that we will discuss further. The DM density may saturate at the very centre of a structure because of annihilation, and a cut-off radius is usually set by equating the gravitational infall rate with the WIMP annihilation rate, of which the typical size is $\sim 10^{-6}$ pc for usual WIMPs. For the smooth component, we choose $r_s = 20$ kpc, and we normalise the density profile $\rho_{\rm sm}$ with respect to the local DM density in the solar neighbourhood $\rho_{\rm sm}(R_\odot = 8\text{ kpc}) = \rho_\odot = 0.3\text{ GeV/cm}^3$. The inner shape of the smooth component has no significant impact on the DM annihilation induced CRs because of propagation effects that strongly dilute what originates from the central regions of the Galaxy. Fluxes at the Earth are much more sensitive to modifications of local distribution of DM. Finally, we set the Galactic virial radius to $R_{\rm vir}^{\rm h}$= 280 kpc, so that the total halo mass inside $R_{\rm vir}^{\rm h}$ is $M_{\rm vir}^{\rm h} = 1.12 \times 10^{12} M_\odot$. Sub-halos are found to be more and more numerous as the resolution of N-body simulations gets thinner and thinner. For instance [13] found $\sim 10^{15}$ substructures of Earth-mass and of solar system size, in a galaxy-size box at early stages of structure formation ($z = 26$). Such small masses are consequent of very small free streaming scales for the dark matter, and are rather generic for WIMPs. However, one can wonder whether or not such small structures may survive tidal effects and encounter events that characterise their evolution in the host halo. While this issue is still debated, we have scrutinised the effect of considering different minimal masses — 10^{-6}, 1 and 10^6 $M_\odot$— in the original paper. A complete sub-halo phase space may be portrayed with the following normalised probability density functions

(pdf), which define the number density of sub-halos per mass unit in the MW: $\frac{dn_{\rm cl}}{dM}(\vec{x},M) = \frac{dN_{\rm cl}}{dV\,dM} = N_0 \times \frac{d\mathcal{P}_V(\vec{x})}{dV} \times \frac{d\mathcal{P}_M(M)}{dM}$, where we assume that the mass function has no spatial dependence. The mass function is usually found to be a power law ; we therefore define: $\frac{d\mathcal{P}_M(M)}{dM} \equiv K_M M^{-\alpha_{\rm m}}$, where K_M normalises the pdf to unity in the mass range used for sub-halos, so that it depends on $M_{\rm min}$, $M_{\rm max}$ and $\alpha_{\rm m}$. For simplicity, we have considered the same power law all over the mass range, which is not that obvious but motivated by theoretical arguments [15] and simulation results [13]. Regarding the spatial pdf, we have retained two cases. The first one is that sub-halos spatially stick to the smooth host DM halo density profile, which may be an appropriate description for very light clumps, as they may behave like test particles. In that case, $d\mathcal{P}_V(\vec{x})/dV = \rho_{\rm sm}(\vec{x})/M^{\rm h}_{\rm vir}$, which is normalised inside $R^{\rm h}_{\rm vir}$. Nevertheless, according to the current results of N-body simulations for which the resolved sub-halo mass is $\gtrsim 10^6 M_\odot$ (*cf.* e.g. the *Via Lactea* [14]), the spatial distribution of substructures is mostly anti-biased compared to the smooth component. This is characterised by a spherical isothermal distribution with a core radius r_c roughly equal to the scale radius r_s of the MW. This is the 2$^{\rm nd}$ case: $\frac{d\mathcal{P}_V(r)}{dV} = K_V \times \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-1}$, where K_V normalises the distribution to unity within the virial radius $R^{\rm h}_{\rm vir}$ of the MW. The total number of sub-halos N_0 depends on $\alpha_{\rm m}$ and $M_{\rm min}$, but there are constraints from current N-body simulations. Indeed, there is a rather good agreement about the counting of well resolved sub-halos inside a MW-type host: ~ 100 objects are found in the mass range $10^8 - 10^{10} M_\odot$. Therefore, we ask for $N_0 \times \int_{10^8 M_\odot}^{10^{10} M_\odot} d\mathcal{P}_M/dM = 100$. The inner profile $\rho_{\rm cl}$ of any sub-halo is also described with the NFW or the Moore models, the latter defining a maximally optimised scenario. Besides, we need additional prescriptions in order to set the scale radius r_s and the associated scale density ρ_s for sub-halos. The standard method is to define some reference quantities and to connect them with the *physical* scale variables by some functions usually fitted on the N-body simulation results. Those reference quantities are the *virial* mass and radius, which are related through: $M_{\rm vir} = \frac{4\pi}{3} {R_{\rm vir}}^3 \times (\Delta_{\rm vir}(z=0)\Omega_M(z=0)\rho_{\rm crit}(z=0))$. Such a definition is not physical in the sense that any mass is related to a radius enclosing $\Delta_{\rm vir}$ times the background matter density today (we set $\Delta_{\rm vir}$=340). Indeed, sub-halos do not evolve in a constant background, but inside host halos where the density is not the background density. Moreover, the previous equation does not carry the formation history of the structure, e.g. the fact that lighter clumps should be denser (because formed in a denser universe). This is actually encoded in the *concentration* parameter $c_{\rm vir}$, which connects the virial radius, as defined above from the mass, to the more physical scale radius: $c_{\rm vir} = \frac{R_{\rm vir}}{r_{-2}}$, where r_{-2} is the radius at which $d/dr\left(r^2\rho_{\rm cl}(r)\right)|_{r=r_{-2}} = 0$ ($r_{-2} = r_s$ for an NFW profile).

There are different concentration models in the literature, and we have taken two extreme cases that encompass them (i) the — maximal — Bullock et al [16] model (B01) and (ii) the — minimal — Eke et al [17] model (ENS01). Once the sub-halo mass and the mass-concentration relation are known, all the clump properties are fixed, and one can compute the corresponding annihilation rate. As it is proportional to the squared DM sub-halo density, it is useful to define an *effective annihilation volume* for any sub-halo as follows: $\xi \equiv \int_{\rm cl} d^3\vec{x} \left(\frac{\rho_{\rm cl}}{\rho_\odot}\right)^2$, where $\rho_\odot$, the local dark matter density, allows for a normalisation to the local annihilation rate. Such an effective volume is that of the DM within the clump would have if it were diluted down to the density $\rho_\odot$. It is convenient to normalise to local quantities because cosmic ray propagation favours a local origin for the exotic contribution. Therefore, $\xi = f(M)$, and the mass pdf $d\mathcal{P}_M/dM$ translates very simply to an annihilation rate pdf. Changing the inner profile results in a shift by a constant factor, e.g. $\xi_{\rm moore} \simeq 10 \times \xi_{\rm nfw}$, whereas changing the concentration model is not as straightforward. Nevertheless, we find very roughly that $\xi_{\rm B01} \simeq 1 - 10 \times \xi_{\rm ENS01}$, depending on the sub-halo mass.

Cosmic ray propagation: The CR propagation modelling is a key ingredient for this kind of studies. Here, we adopt a slab diffusion zone, featured by its radial extension that we fix to $R_{\rm slab} = 30$ kpc, and by its half-thickness L, 3 kpc here. CRs are either confined within the slab or escape forever, which is merely fulfilled by imposing Dirichlet boundary conditions to the diffusion equation (the CR number density vanishes on the borders). Regarding the transport processes, the spatial independent diffusion coefficient is given by $K(E) = \beta K_0 \mathcal{R}^\delta$ (where $\mathcal{R} = pc/Ze$ is the rigidity) and a constant convective wind $V_{\rm conv}$ is directed outwards along the vertical axis. Such a configuration is quantified with the *medium* set of parameters provided by [18]: $K_0 = 0.0112$ kpc^2/Myr, $\delta = 0.7$ and $V_{\rm conv} = 12$ km/s. One can easily write and solve the diffusion equations for both $e^{+\prime}$s and $\overline{p}'$s for this kind of geometry. For $e^{+\prime}$s, the main processes that come into play are the energy losses (mainly inverse Compton diffusion off CMB or IR photons, and synchrotron radiation), and the diffusion on the magnetic turbulences. Disregarding the convection process, which is much less efficient than energy losses, one can express the typical propagation length for $e^{+\prime}$s as: $\lambda_{\rm d} \equiv \left(2K_0\tau_E\left(\frac{\epsilon^{\delta-1}-\epsilon_S^{\delta-1}}{1-\delta}\right)\right)^{1/2}$, where $\epsilon \equiv E/\{E_0 = 1 \text{ GeV}\}$. Assuming an infinite 3D spherical diffusion zone (correct while $\lambda_{\rm d} \lesssim L$), the e^+ propagator is proportional to a Gaussian function of the source distance with $\sigma = \lambda_{\rm d}$ ($\lesssim$ few kpc): Sources located farther than $\lambda_{\rm d}$ will almost not contribute to the flux at the Earth. λ_d being a decreasing function of the detected energy, the effective volume in which $e^{+\prime}$s propagate increases as they loose energy. The propagation of $\overline{p}'$s must include spallation processes and wind convection that occur in the thin Galactic disc, so we can

not use a simple spherical symmetry to derive a global expression. Moreover, the energy losses are negligible for this species, which modifies significantly the picture that we had for e^+'s. Nevertheless, it is useful to write, as for e^+'s, the typical propagation length: $\Lambda_\mathrm{d} \equiv \frac{K(E)}{V_\mathrm{conv}}$, where convection is assumed to dominate over spallation in average, which is correct unless at sub-GeV energies. This is quite different from e^+'s because this length is an increasing function of energy. The picture is therefore reversed, and the propagation volume is much larger at higher energy for $\overline{p}$'s (Λ_d reaches the size of the diffusion slab at energies of order 10-100 GeV). We now define a convenient Green function for any CR species, that encodes the injected spectrum induced by dark matter annihilation: $\tilde{\mathcal{G}}(E, \vec{x}_\odot \leftarrow \vec{x}) \equiv \int_E^{E_\mathrm{max}} dE_S \; \mathcal{G}(E, \vec{x}_\odot \leftarrow E_S, \vec{x}) \times \frac{dN_\mathrm{CR}(E_S)}{dE_S}$, where dN_CR/dE_S is the injected spectrum at source ($E = E_S$ for $\overline{p}$'s).

Exotic fluxes and associated boost factors: The Galactic host halo is described by a smooth dark matter distribution ρ_sm, so that the corresponding primary CR flux reads: $\phi_\mathrm{sm}(E) = \frac{v}{4\pi}\mathcal{S}\int_\mathrm{halo} d^3\vec{x} \;\; \tilde{\mathcal{G}}(E, \vec{x}_\odot \leftarrow \vec{x}) \left(\frac{\rho_\mathrm{sm}}{\rho_\odot}\right)^2$ where v is the CR velocity and $\mathcal{S} \equiv \delta \langle \sigma_\mathrm{ann} v \rangle {\rho_\odot}^2/(2{m_\chi}^2)$ encodes the main WIMP properties[2]. Sub-halos can be considered as point-like sources, and the flux due to the i^th object is merely: $\phi_{\mathrm{cl},i}(E) = \frac{v}{4\pi} \times \mathcal{S} \times \xi_i \times \tilde{\mathcal{G}}(E, \vec{x}_\odot \leftarrow \vec{x}_i)$. Then, we have to sum over the whole population. We can derive a statistical prediction by integrating over the sub-halo phase space. The overall clump contribution is thus: $\phi_\mathrm{cl,tot}(E) = \frac{v}{4\pi} N_0 \int dM \xi(M) \frac{d\mathcal{P}_M(M)}{dM} \times \int d^3\vec{x} \tilde{\mathcal{G}}(E, \vec{x}_\odot \leftarrow \vec{x}) \frac{d\mathcal{P}_V(\vec{x})}{d^3\vec{x}} = N_0 \langle \phi_\mathrm{cl} \rangle = \frac{v}{4\pi} \mathcal{S} N_0 \langle \xi \rangle \langle \tilde{\mathcal{G}} \rangle$ where the 1^st equality takes directly the — normalised — pdfs into account (this limit would be reached e.g. for an infinite number of MC realisations). Those pdfs characterise ξ and $\tilde{\mathcal{G}}$, the former being a function of the sub-halo mass and the latter being spatially weighted with $d\mathcal{P}_V/dV$. The last equality gives the same quantities in terms of statistical mean values. This assumes no correlations between the considered variables. The calculation of the variance of CR fluxes originating from sub-halos $\sigma_\mathrm{cl,tot}$ is straightforward: $\frac{\sigma^2_\mathrm{cl,tot}}{\phi^2_\mathrm{cl,tot}} = \frac{1}{N_0}\left(\frac{\sigma^2_\xi}{\langle \xi \rangle^2} + \frac{\sigma^2_{\tilde{\mathcal{G}}}}{\langle \tilde{\mathcal{G}} \rangle^2} + \frac{\sigma^2_\xi \sigma^2_{\tilde{\mathcal{G}}}}{\langle \xi \rangle^2 \langle \tilde{\mathcal{G}} \rangle^2} \right)$, where σ_x is the variance corresponding to any variable x. The *boost factors* for e^+'s and $\overline{p}$'s are the ratios of the CR fluxes originating from a clumpy halo to those calculated for the host smooth halo alone. Normalising the whole dark matter average density profile at the Earth, the effective boost is: $B_\mathrm{eff}(E) = (1 - f_\odot)^2 + \phi_\mathrm{cl,tot}/\phi_\mathrm{sm}$, which depends on energy. $f_\odot$ is the average local DM density in form of clumps (the smooth component density is $(1 - f_\odot)\rho_\mathrm{sm}$ when sub-halos are added). Estimates of the mean fluxes for e^+'s and $\overline{p}$'s and associated statistical variances have been

[2] $\delta = 1/2$ for Majorana particles, 1 otherwise.

performed by using the semi-analytical method proposed by [2]. In order to exhaustively characterise the effect of all the relevant variables in the problem, we have varied the minimal mass of sub-halos $M_{\rm min}$, the logarithmic slope of the mass function $\alpha_{\rm m}$, the concentration model, the inner profiles of sub-halos, their spatial distribution in the MW ; we have also varied the CR propagation model. This provides a way to anticipate predictions for any clumpiness modelling, and is also useful to sketch some theoretical uncertainty contours. The full results are available in the original paper [1]. In this proceeding, we quickly discuss three cases: *minimal*, *reference* and *maximal* models, considering a medium set of CR propagation parameters. By defining a model with $\{M_{\rm min}, \alpha_{\rm m}$, inner profile, concentration model, spatial distribution$\}$, the *minimal* reads $\{10^6 M_\odot$,1.8,NFW,ENS01,cored $\}$, the *reference* $\{10^{-6} M_\odot$,1.9,NFW,B01,cored $\}$ and the *maximal* $\{10^{-6} M_\odot$,2.0,Moore,B01,smooth-tracking $\}$. The most optimistic — *maximal* — case, which is also the most unlikely, yields boost factors ~ 20, which depend on the CR energy, while the other configurations do not permit substantial enhancement, the effective boost factors $B_{\rm eff}$ remaining close to unity for both $e^{+\prime}$s and $\overline{p}'$s. Taking back NFW profiles in the *maximal* setup, we would obtain $\sim 20/10 = 2$. The variance affecting those predictions decreases with the local average number density of sub-halos. Indeed, the statistical variance depends on the number of objects located inside a volume bounded by the CR propagation length and contributing to the flux at the Earth. As the propagation length is a function of the CR energy which depends on the species — increasing (respectively decreasing) with energy for $\overline{p}'$s ($e^{+\prime}$s) — one can understand why the picture can be different for different CR species. As a very general statement, the variance on primary fluxes is smaller at lower energy for $e^{+\prime}$s, or at higher energy for $\overline{p}'$s. Nevertheless, even with a large variance for sub-halo fluxes, the statistical error will still be small for boost factors as soon as the smooth contribution will dominate (i.e. errors are small for small boost factors).

Conclusion: In [1], we have tried to exhaustively tackle the problem of DM clumpiness effects in the frame of indirect detection of annihilating DM with antimatter cosmic rays. We have taken the whole phase space of sub-halos into account, using a compilation of results found in the literature, coming from N-body simulations as well as from analytical models. We have sketched the theoretical uncertainties affecting the useful dark matter parameters, and defined rather wide associated ranges to account for them. For the sake of completeness, we have also considered different sets of CR propagation parameters, which are still degenerate and not enough constrained by the existing secondary/primary CR measurements. We refer the reader to the original paper for very detailed results. In this proceeding, we have only illustrated the cases of extreme configurations, which can provide boost factors $\lesssim 20$ for both

$e^{+\prime}$s and $\overline{p}'$s. Nevertheless, our analysis strongly disfavours large and even mildly boost factors for most parts of the large parameter space used to describe sub-halos. This is almost independent of CR propagation uncertainties.

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